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Soluções Listas 2 e 3

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Solução

Lista 2 [illegible] 16/04/15

(1)

a) $y' = \frac{x^2}{y} \therefore \frac{dy}{dx} = \frac{x^2}{y}$ $\int y\,dy = \int x^2\,dx \therefore$

$\therefore \frac{y^2}{2} = \frac{x^3}{3} + C$ $\boxed{y = \pm\sqrt{\frac{2x^2}{3} + C}}$

b) $y' + y^2 \operatorname{sen} x = 0 \Rightarrow \frac{dy}{dx} = -y^2 \operatorname{sen} x \therefore \int \frac{1}{y^2}\,dy = -\int \operatorname{sen} x\,dx$

$\frac{y^{-1}}{-1} = \cos x + C$ $\frac{1}{y} = -\cos x + C$ $\boxed{y = -\frac{1}{\cos x + C}}$

c) $\frac{dy}{dx} = \frac{3x^2 - 1}{3 + 2y} \therefore \int (3 + 2y)\,dy = \int (3x^2 - 1)\,dx$

$3y + \frac{2y^2}{2} = \frac{3x^3}{3} -$ [redacted] $x + C$

$3y + y^2 = x^3 -$ [redacted] $x + C$

$\boxed{y(y+3) = x^3 - \text{[redacted]}\, x + C}$ Implícita

$$d)\ y' = \cos^2 x \cdot \cos^2(2y) \qquad \int \cos^2(2y)^{-1}\,dy = \int \cos^2(x)\,dx \quad \text{(I)}$$

Tabelas de integrais: $\int \cos^2(x)\,dx = \frac{1}{2}x + \frac{1}{4}\text{sen}\,2x + C$

$$\int \sec^2(2y)\,dy = 1/2tg(2y)$$

Substituindo em I

$$1/2tg(2y) = \frac{1}{2}x + \frac{1}{4}\text{sen}(2x) + C$$

$$2y = \text{Arctg}\left(\quad x + \frac{1}{2}\text{sen}(2x) + C\right)$$

$$\boxed{y(x) = \frac{1}{2}\text{Arctg}\left(\quad x + \frac{1}{2}\text{sen}(2x) + C\right)}$$

$$e)\ \frac{dy}{dx} = \frac{(x - e^{-x})}{(y + e^y)} \quad \therefore \quad \int (y + e^y)\,dy = \int (x - e^{-x})\,dx$$

$$\int y\,dy + \int e^y\,dy = \int x\,dx - \int e^{-x}\,dx$$

$$\boxed{\frac{y^2}{2} + e^y = \frac{x^2}{2} + e^{-x} + C}$$

$u = -x$

$\frac{du}{dx} = -1 \qquad dx = -du$

$-\int e^u\,du = -e^u$

(2)

$$\frac{dy}{dt} + P(x)y = Q(x)$$

$$d)\ y' - y = 2te^{2t} \quad [y(0)=1] \quad P(t) = -1 \qquad \mu(t) = \exp\left(\int -1\,dt\right) = e^{-t}$$

$$e^{-t}y' - e^{-t}y = 2te^{t}$$

$$\frac{d}{dt}\left(e^{-t}y\right) = 2te^{t} \quad \therefore \quad \int d\left(e^{-t}y\right) = 2\int te^{t}\,dt$$

$$\text{(I)} \quad e^{-t}y = 2\int te^{t}\,dt \qquad \text{integração por partes}$$

$u = t \rightarrow du = dt$

$dv = e^{t}dt \rightarrow v = e^{t}$

$$\int u\,dv = uv - \int v\,du$$

$$\int te^{t}\,dt = te^{t} - \int e^{t}\,dt = te^{t} - e^{t} + C$$

Subst em I

$$e^{-t}y = 2\left(te^{t} - e^{t} + c\right)$$

$$y = 2\left(te^{2t} - e^{2t} + ce^{t}\right) = 2e^{t}\left(te^{t} - e^{t} + c\right)$$

$$\boxed{y(t) = 2e^{t}\left[e^{t}(t-1) + c\right]}$$

$$y(0) = 2e^{0}\left[e^{0}(0-1) + c\right] = 2[-1 + c] = 1$$

$$C = \frac{1}{2} + 1 = \frac{1+2}{2} = \frac{3}{2}$$

(2b) $y' + 2y = te^{-2t} \qquad y(1) = 0$

$p(t) = 2 \qquad \mu(t) = e^{\int 2dt} = e^{2t}$

$e^{2t}y' + 2e^{2t}y = t$

$\frac{d}{dt}\left(e^{2t}y\right) = t \; \therefore \int d\left(e^{2t}y\right) = \int t\,dt$

$e^{2t}y = \frac{t^2}{2} + C \; \therefore \; y(t) = \frac{1}{e^{2t}}\left(\frac{t^2}{2} + C\right)$

$y(1) = \frac{1}{e^2}\left(\frac{1}{2} + C\right) = 0 \; \therefore \; C = -\frac{1}{2}$

$y(t) = e^{-2t}\left(\frac{t^2}{2} - \frac{1}{2}\right) = \frac{e^{-2t}}{2}\left(t^2 - 1\right)$

(2c) $ty' + 2y = t^2 - t + 1 \qquad y(1) = 1/2$

$\div t$

$y' + \frac{2}{t}y = t - 1 + \frac{1}{t}$ (Forma Padrão)

$p(t) = \frac{2}{t} \qquad \mu(t) = e^{\int \frac{2}{t}dt} = e^{2\ln t} = t^2$

$t^2y' + 2ty = t^3 - t^2 + t$

$\frac{d}{dt}\left(t^2y\right) = t^3 - t^2 + t \; \therefore \int d\left(t^2y\right) = \int \left(t^3 - t^2 + t\right)dt$

$t^2y = \frac{t^4}{4} - \frac{t^3}{3} + \frac{t^2}{2} + C$

$y(t) = \frac{t^2}{4} - \frac{t}{3} + \frac{1}{2} + Ct^{-2}$

$y(1) = \frac{1}{4} - \frac{1}{3} + \frac{1}{2} + C = \frac{1}{2} \qquad C = \frac{1}{3} - \frac{1}{4} = \frac{4-3}{12} = \frac{1}{12}$

$y(t) = \frac{t^2}{4} - \frac{t}{3} + \frac{1}{2} + \frac{t^{-2}}{12}$ ✓

Explica $t > 0$

(2d)

$y' + \frac{2}{t}y = \frac{\cos(t)}{t^2} \qquad y(\pi) = 0$

$p(t) = \frac{2}{t} \qquad \mu(t) = e^{\int \frac{2}{t}dt} = e^{2\ln t} = e^{\ln t^2} = t^2$

$t^2y' + 2ty = \cos(t)$

$\frac{d}{dt}\left(t^2y\right) = \cos(t) \; \therefore \int d\left(t^2y\right) = \int \cos(t)\,dt$

$t^2y = \text{sen}(t) + C$

$y(t) = t^{-2}\left[\text{sen}(t) + C\right]$

$y(\pi) = (\pi)^{-2}\left[\text{sen}(\pi) + C\right] = 0 \; \therefore \; \boxed{C = 0}$

$\boxed{y(t) = \frac{1}{t^2}\,\text{sen}(t)}$

2e) $t^3y' + 4t^2y = e^{-t}$ $\quad y(-1)=0 \quad t>0$

$y' + \frac{4}{t}y = t^{-3}e^{-t}$

$p(t) = \frac{4}{t} \rightarrow \mu(t) = e^{\int \frac{4}{t}dt} = e^{4\ln t} = e^{\ln t^4} = t^4$

$t^4y' + 4t^3y = te^{-t}$

$\frac{d}{dt}(t^4y) = te^{-t} \therefore \int d(t^4y) = \int te^{-t}dt$ (a) (I)

(a) $\int te^{-t}dt \quad u=t \quad du=dt \quad dv = e^{-t}dt \quad v=-e^{-t}$ $\quad \int e^{-t}dt \rightarrow -\int e^{g}dg = -e^{-t}$ $\quad g=-t \quad dg=-dt$

$\int te^{-t}dt = uv - \int v\,du = -te^{-t} - \int(-e^{-t})dt$

$= -te^{-t} + \int e^{-t}dt = -te^{-t} - e^{-t} + C$ (II)

Subst II em I

$t^4y = -te^{-t} - e^{-t} + C$

$y(t) = -t^{-3}e^{-t} - t^{-4}e^{-t} + t^{-4}C$

$y(-1) = -(-1)^{-1}e^{1} - (-1)^{-1}e^{1} + (-1)^{-1}C = 0$

$= e^{1} - e^{1} + C = 0 \therefore C = 0$

$y(t) = t^{-3}e^{-t}(-1 - t^{-1})$

d f) $ty' + 2y = \text{sen}\,t$ $\quad y(\pi/2) = 1,\ t>0$

$y' + \frac{2}{t}y = \frac{\text{sen}\,t}{t}$

$p(t) = \frac{2}{t} \quad \mu(t) = e^{\int \frac{2}{t}dt} \quad e^{\ln t^2} = t^2$

$t^2y' + 2ty = t\,\text{sen}\,t$

$\frac{d}{dt}(t^2y) = t\,\text{sen}\,t \qquad \int d(t^2y) = \int t\,\text{sen}\,t\,dt$ (I)

(a) $\int t\,\text{sen}(t)dt = uv - \int v\,du = -t\cos(t) - \int(-\cos(t))dt$

$u=t \quad du=dt \qquad = -t\cos(t) + \text{sen}(t) + C$ (II)

$dv = \text{sen}(t) \quad v = -\cos(t)$

Subst II em I

$t^2y = -t\cos(t) + \text{sen}(t) + C$

$y(t) = -\frac{\cos(t)}{t} + \frac{\text{sen}(t)}{t^2} + \frac{C}{t^2}$

$y(\frac{\pi}{2}) = -\frac{0}{\pi/2} + \frac{1}{(\pi/2)^2} + \frac{C}{(\pi/2)^2} = 1 \therefore C = (\frac{\pi}{2})^2 - 1$

$y(t) = \frac{1}{t^2}\left[-t\cos(t) + \text{sen}(t) + (\pi/2)^2 - 1\right]$

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$y = x\,v(x) \therefore \frac{dy}{dx} = x v'(x) + \dot{x} v(x) = x\frac{dv}{dx} + v$

1a) $y' = \frac{x^2+xy+y^2}{x^2} \frac{\div x^2}{\div x^2} = 1 + \frac{y}{x} + \left(\frac{y}{x}\right)^2$ empregando a substituição

$\cancel{v} + x\frac{dv}{dx} = 1 + \cancel{v} + v^2$

$x\frac{dv}{dx} = 1 + v^2 \therefore \int \frac{1}{(1+v^2)}\,dv = \int \frac{1}{x}\,dx$ tg a = 1; arctg(1) = a

$$\text{arctg}(v) = \ln|x| + C$$

$\text{arctg}\left(\frac{y}{x}\right) = \ln|x| + C \therefore \text{tg}(\ln|x| + C) = \frac{y}{x}$

$\boxed{y(x) = x\,\text{tg}(\ln|x| + C)}$

1b) $\frac{dy}{dx} = \frac{x^2+3y^2}{2xy} \frac{\div x^2}{\div x^2} = \frac{1+3(y/x)^2}{2(y/x)}$ Empregando a substituição indicada

$v + x\frac{dv}{dx} = \frac{1+3v^2}{2v}$. $x\frac{dv}{dx} = \frac{1+3v^2-2v^2}{2v}$

$\int\frac{2v}{1+v^2}\,dv = \int \frac{1}{x}\,dx \therefore \ln(v^2+1) = \ln(x) + C$

$v^2 + 1 = Cx$ $\left(\frac{y}{x}\right)^2 = Cx - 1 \therefore \boxed{y(x) = x\sqrt{Cx-1}}$

1c) $\frac{dy}{dx} = \frac{4y-3x}{2x-y} \frac{\div x}{\div x}$

$\frac{dy}{dx} = \frac{4y/x - 3}{2 - y/x}$ Usando a substituição

$v + x\frac{dv}{dx} = \frac{4v-3}{2-v} \therefore x\frac{dv}{dx} = \frac{4v-3-2v+v^2}{2-v}$

$x\frac{dv}{dx} = \frac{v^2+2v-3}{2-v} \therefore \int \frac{2-v}{v^2+2v-3}\,dv = \int\frac{1}{x}\,dx$ (I)

$\int\frac{2-v}{v^2+2v-3}\,dv$ Método frações parciais $v^2+2v-3=0$ $v = 1, -3$

$\frac{2-v}{(v-1)(v+3)} = \frac{A}{(v-1)} + \frac{B}{(v+3)} \therefore 2 - v = A(v+3) + B(v-1)$

$2 - v = Av + 3A + Bv - B = v(A+B) + (3A-B)$

$A + B = -1 \to A = -1 - B$

$3A - B = 2 \to -3 - 3B - B = 2 \therefore B = -\frac{5}{4}$ $A = -1 + \frac{5}{4} = \frac{-4+5}{4} = \frac{1}{4}$

$\int\frac{2-v}{v^2+2v-3}\,dv = \int\frac{1}{4(v-1)}\,dv - \frac{5}{4}\int\frac{1}{(v+3)}\,dv = \frac{1}{4}\ln(v-1) - \frac{5}{4}\ln(v+3)$

Subst em (I)

$\frac{1}{4}\ln(v-1) - \frac{5}{4}\ln(v+3) = \ln(x) + C$

$\ln(v-1) - \ln(v+3)^5 = \ln(x)^4 + C$

$\ln \frac{(v-1)}{(v+3)^5} = \ln x^4 + c \quad \therefore \quad \frac{(v-1)}{(v+3)^5} = Cx^4$

$\frac{(y/x - 1)}{(y/x+3)^5} = Cx^4$

2a) $(2x+3) + (2y-2)y' = 0$

$M = 2x+3 \quad M_y = 0$ É exata
$N = 2y-2 \quad N_x = 0$

$\int M dx = \int (2x+3)\,dx = \frac{2x^2}{2} + 3x + g(y) = \psi(x,y)$ (I)

$\frac{\partial \psi}{\partial y} = N = g'(y) = 2y-2 \qquad g = \int (2y-2)\,dy$

$g = \frac{2y^2}{2} - 2y$ (II)

Subst. II em I

$\psi(x,y) = x^2 + 3x + y^2 - 2y$

Como $\frac{d\psi}{dx} = 0 \quad \psi = C$ Assim

$\boxed{x^2 + 3x + y^2 - 2y = C}$

2b) $(2x+4y) - (2x-2y)y' = 0$

$M = 2x+4y \rightarrow M_y = 4$ Não exata!
$N = 2x-2y \rightarrow N_x = 2$

2c) $(3x^2 - 2xy + 2)dx + (6y^2 - x^2 + 3)dy = 0$

$M = 3x^2 - 2xy + 2 \quad M_y = -2x$ É exata!
$N = 6y^2 - x^2 + 3 \quad N_x = -2x$

$\int M dx = \int (3x^2 - 2xy + 2)\,dx = \frac{3x^3}{3} - \frac{2x^2y}{2} + 2x + g(y) = \psi(x,y)$ (I)

$\frac{\partial \psi}{\partial y} = N = -x^2 + g'(y) = 6y^2 - x^2 + 3$

$g'(y) = 6y^2 - x^2 + 3 + x^2$

(II) $g(y) = \int g'(y)\,dy = \int (6y^2 + 3)\,dy = \frac{6y^3}{3} + 3y$

Subst. (II) em (I)

$\psi(x,y) = x^3 - x^2y + 2x + 2y^3 + 3y$

$= x^3 + 2y^3 - x^2y + 2x + 3y = C \qquad \left(\frac{d\psi}{dx} = 0\right)$

$\frac{\partial \psi}{\partial x} = 3x^2 - 2xy + 2 = M$

$\frac{\partial \psi}{\partial y} = 6y^2 - x^2 + 3 = N$

2d) $(ax+by)+(bx+cy)y'=0$

$M=(ax+by) \quad M_y=b \quad$ Exata!

$N=(bx+cy) \quad N_x=b$

$$\int M\,dx=\int (ax+by)\,dx=\frac{ax^2}{2}+bxy+g(y)=\psi(x,y) \quad \mathrm{I}$$

$$\frac{\partial \psi}{\partial y}=N=bx+g'(y)=bx+cy$$

$$g(y)=\int g'(y)\,dy=\int cy\,dy=\frac{cy^2}{2} \quad \mathrm{II}$$

Subst II em I

$$\psi(x,y)=\frac{ax^2}{2}+\frac{cy^2}{2}+bxy=\text{CTE}$$

2e) $(ax-by)+(bx-cy)y'=0$

$M=(ax-by) \quad M_y=-b \quad M_y\neq N_x \quad$ Não é exata!

$N=(bx-cy) \quad N_x=b$

3a) $x^2y^3+x(1+y^2)y'=0$

$M=x^2y^3 \quad M_y=3x^2y^2 \quad$ Não é exata.

$N=x(1+y^2) \quad N_x=1+y^2$

Como $\mu(x)$ e $\mu(y)$ $\nexists$ (verifique) simplifica-se a equação diferencial através da $\div$ por "x"

$$xy^3 + (1+y^2)y' = 0 \qquad \mathrm{I}$$

$M = xy^3 \rightarrow M_y = 3xy^2$

$N = (1+y^2) \rightarrow N_x = 0$

$\mu(x)$ $\dfrac{M_y - N_x}{N} = \dfrac{3xy^2 - 0}{(1+y^2)}$ logo $\mu(x) \nexists$

$\mu(y)$ $\dfrac{N_x - M_y}{M} = \dfrac{0 - 3xy^2}{xy^3} = \dfrac{-3}{y}$ logo $\mu(y) \exists$

$$\mu(y) = e^{\int -\frac{3}{y} dy} = e^{\ln y^{-3}} = \frac{1}{y^3}$$

Multiplicando I pelo fator integrante

$$x + \frac{(1+y^2)}{y^3}y' = 0$$

$\mu M = x \qquad (\mu M)_y = 0$ É exata!

$\mu N = \dfrac{(1+y^2)}{y^3} \qquad (\mu N)_x = 0$

$$\int \mu M dx = \int x dx = \frac{x^2}{2} + g(y) = \Psi(x,y)$$

$\dfrac{\partial \Psi}{\partial y} = \mu N = g'(y) = \dfrac{(1+y^2)}{y^3}$ $\qquad g(y) = \int g'(y) dy = \int \dfrac{1}{y^3} dy + \int \dfrac{1}{y} dy$

$$g(y) = \frac{y^{-2}}{-2} + \ln|y|$$

$$\Psi(x,y) = \frac{x^2}{2} - \frac{1}{2y^2} + \ln|y| = C$$

$$3b)\ y\,dx + (2x - ye^y)dy = 0 \qquad \mathrm{I}$$

$M = y \qquad M_y = 1$ Não é exata.

$N = (2x - ye^y) \qquad N_x = 2$

$\mu(x)$ $\dfrac{M_y - N_x}{N} = \dfrac{1-2}{(2x - ye^y)}$ $\qquad \mu(x) \nexists$

$\mu(y)$ $\dfrac{N_x - M_y}{M} = \dfrac{2-1}{y} = \dfrac{1}{y}$ $\mu(y) \exists$

$$\mu(y) = e^{\int \frac{1}{y} dy} = y$$

Multiplicando I por $\mu(y) = y$

$$y^2 dx + (2xy - y^2 e^y)dy = 0$$

$(\mu M) = y^2 \qquad (\mu M)_y = 2y$

É exata!

$(\mu N) = 2xy - y^2 e^y \qquad (\mu N)_x = 2y$

$$\int \mu M dx = \int y^2 dx = xy^2 + g(y) = \Psi(x,y)$$

$$\frac{\partial \Psi}{\partial y} = (\mu N) = 2xy + g'(y) = 2xy - y^2 e^y$$

$$g(y) = \int g'(y) dy = \int -y^2 e^y dy = -e^y(y^2 - 2y + 2)$$

$$\Psi(x,y) = xy^2 - e^y(y^2 - 2y + 2) = C$$

Prof. Diog

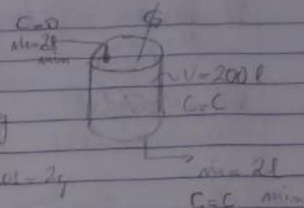
Aplicações:

4) $V = 200\,l$

$C_0 = 1\,g/l \Rightarrow m_0 = V \cdot C_0 = 200 \cdot 1 = 200\,g$

$t = ?$

$C_f = 0{,}01\,C_0 = 0{,}01\,g/l \quad m_f = 200 \cdot 0{,}01 = 2\,g$



$$\boxed{\frac{dm}{dt} = \dot{m}_e - \dot{m}_s}$$

Conservação da massa

$\dot{m}_e = \frac{2l}{min} \cdot \frac{0\,g}{l} = 0 \left[\frac{g}{min}\right]$

$\dot{m}_s = \frac{2l}{min} \cdot \frac{m}{200}\left[\frac{g}{l}\right] = \frac{m}{100}\left[\frac{g}{min}\right]$

$$\frac{dm}{dt} = 0 - \frac{m}{100}$$

$$\frac{dm}{dt} = -\frac{m}{100} \quad \therefore \quad \int \frac{1}{m}\,dm = -\frac{1}{100}\int dt \quad \therefore \quad \ln|m| = -\frac{1}{100}t + C_1$$

$$m(t) = C\exp(-0{,}01t)$$

$$m(0) = C\exp(-0{,}01 \cdot 0) = 200\,g \qquad C = 200\,g$$

$$m(t) = 200\exp(-0{,}01t)$$

$$2 = 200\exp(-0{,}01t) \quad \therefore \quad t = 460{,}5\ min$$

5) $\frac{dS}{dt} = rS \qquad \int \frac{1}{S}\,dS = r\,dt \qquad \ln|S| = rt + C_1$

$S(t) = C\exp(rt) \qquad S(0) = S_0 \quad \therefore \quad C = S_0$

$S(t) = S_0\exp(rt)$

a) $t = ? \qquad S = 2S_0$

$2S_0 = S_0\exp(rt) \quad \therefore \quad \ln(2) = rt \quad \therefore \quad t = \frac{\ln(2)}{r}$

b) $t = \frac{\ln(2)}{0{,}07\,[1/ano]} = 9{,}9$ anos

6) $\frac{dS}{dt} = rS + k \rightarrow \frac{dS}{dt} - rS = k$ EDO linear 1ª ordem

$$S(t) = S_0 e^{rt} + \left(\frac{k}{r}\right)(e^{rt} - 1)$$

a) Como não há capital inicial $S_0 = 0$, então

$$S(t) = \left(\frac{k}{r}\right)(e^{rt} - 1)$$

b) $r = 0{,}075 \qquad 10^6 = \left(\frac{k}{0{,}075}\right)\left(e^{0{,}075 \cdot 40} - 1\right)$

$S = 1 \cdot 10^6$

$t = 40$ anos $\qquad k = 3929{,}7$ R\$/ano

$k = ?$

c) $k = 2000$ R\$/ano

$S = 10^6$ R\$

$t = 40$ anos

$10^6 = \left(\frac{2000}{r}\right)\left(e^{r\,40} - 1\right)$

$r = 0{,}0977 = 9{,}77\%$

3)

$$\frac{du}{dt} = -k\,[u - T(t)]$$

$u(t)$

$T(t) = T_0 + T_1 \cos(wt)$

a) $\frac{du}{dt} + ku = +kT(t)$

$\frac{du}{dt} + ku = k\,[T_0 + T_1 \cos(wt)]$ (I) EDO linear 1ª ordem

$p(t) = k \qquad \mu(t) = e^{\int p\,dt} = e^{kt}$

Multiplicando os dois lados de (I) por $\mu(t)$

$$e^{kt}\frac{du}{dt} + e^{kt}ku = ke^{kt}\,[T_0 + T_1\cos(wt)]$$

$$\frac{d}{dt}\left(e^{kt}u\right) = ke^{kt}\,[T_0 + T_1\cos(wt)]$$

$$\int d\left(e^{kt}u\right) = k\int e^{kt}\,[T_0 + T_1\cos(wt)]\,dt$$

$$e^{kt}u = T_0 k\int e^{kt}\,dt + T_1 k\int e^{kt}\cos(wt)\,dt + C_1$$

$$e^{kt}u(t) = kT_0\int e^{kt}\,dt + kT_1\int e^{kt}\cos(wt)\,dt + C_1$$

$$e^{kt}u(t) = \frac{kT_0 e^{kt}}{k} + \frac{T_1 k e^{kt}}{(w^2+k^2)}\,[k\cos(wt) + w\,\mathrm{sen}(wt)] + C$$

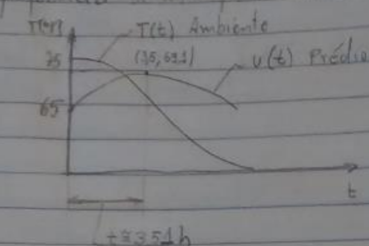
$$u(t) = \underbrace{Ce^{-kt}}_{\text{Parte transiente}} + \underbrace{T_0 + \frac{kT_1\,[k\cos(wt) + w\,\mathrm{sen}(wt)]}{(w^2+k^2)}}_{\text{Estado estacionario}\ S(t)}$$

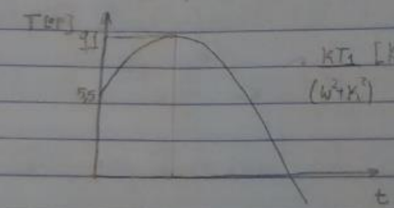
b) $w = \frac{\pi}{12}$ (frequência de 24h para $T(t)$)

$T_0 = 60°F$

$T_1 = 15°F$

$k = 0{,}2\ h^{-1}$





$$\frac{kT_1\,[k\cos(wt) + w\,\mathrm{sen}(wt)]}{(w^2+k^2)}$$

Os valores dos gráficos foram obtidos com auxílio do Maple

Soluções Lista 4

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Solução

Lista 4

1a) $y'' + 2y' - 3y = 0$ $a = 1$, $b = 2$, $c = -3$

$ar^2 + br + c = 0$ (4 casos) reais distintas

$r^2 + 2r - 3 = 0$

$r = \frac{-2 \pm \sqrt{4+12}}{2}$ $r_1 = 1$, $r_2 = -3$

Raízes reais e distintas $y(t) = c_1 e^{r_1 t} + c_2 e^{r_2 t}$

$$\boxed{y(t) = c_1 e^{t} + c_2 e^{-3t}}$$

1b) $y'' + 3y' + 2y = 0$ $a = 1$, $b = 3$, $c = 2$

$ar^2 + br + c = 0$ reais distintas

$r^2 + 3r + 2 = 0$

$r = \frac{-3 \pm \sqrt{9-8}}{2}$ $r_1 = -1$, $r_2 = -2$

$y(t) = c_1 e^{r_1 t} + c_2 e^{r_2 t} = c_1 e^{-t} + c_2 e^{-2t}$

1c) $6y'' - y' - y = 0$ $a = 6$, $b = -1$, $c = -1$

$ar^2 + br + c = 0$ reais e distintas

$6r^2 - r - 1 = 0$

$r = \frac{1 \pm \sqrt{1+24}}{12}$ $r_1 = \frac{1}{2}$, $r_2 = -\frac{1}{3}$

$y(t) = c_1 e^{\frac{1}{2}t} + c_2 e^{-\frac{1}{3}t}$

1d) $2y'' - 3y' + y = 0$ $a = 2$, $b = -3$, $c = 1$

$ar^2 + br + c = 0$ reais e distint[as]

$2r^2 - 3r + 1 = 0$

$r = \frac{3 \pm \sqrt{9-8}}{4}$ $r_1 = 1$, $r_2 = \frac{1}{2}$

$y(t) = c_1 e^{1t} + c_2 e^{\frac{1}{2}t}$

2a) $y_1 = \cos(2t)$, $y_1' = -2\,\mathrm{sen}(2t)$, $y_1'' = -4\cos(2t)$

$y'' + 4y = 0$ (subst)

$-4\cos(2t) + 4\cos(2t) = 0$

$0 = 0$ y_1 é solução!

$y_2 = \mathrm{sen}(2t)$, $y_2' = 2\cos(2t)$, $y_2'' = -4\,\mathrm{sen}(2t)$

$y'' + 4y = 0$ (subst)

$-4\,\mathrm{sen}(2t) + 4\,\mathrm{sen}(2t) = 0$

$0 = 0$ y_2 é solução!

$\frac{y_1}{y_2} = \frac{\cos(2t)}{\mathrm{sen}(2t)} = \frac{1}{tg(2t)} \neq cte$ logo y_1 e y_2 são LI

Portanto, podem ser empregadas para formar uma solução geral do tipo:

$y = c_1 y_1 + c_2 y_2 \therefore y(t) = c_1 \cos(2t) + c_2\,\mathrm{sen}(2t)$

2b) $y_1 = e^t$, $y_1' = e^t$, $y_1'' = e^t$

$y'' - 2y' + y = 0$ (subst)

$e^t - 2e^t + e^t = 0$

$0 = 0$ y_1 é solução

$y_2 = te^t$, $y_2' = e^t + te^t$, $y_2'' = 2e^t + te^t$

$y'' - 2y' + y = 0$ (subst)

$2e^t + te^t - 2e^t - 2te^t + te^t = 0$

$0 = 0$ y_2 é solução

$\frac{y_1}{y_2} = \frac{e^t}{te^t} = \frac{1}{t} \neq cte$ logo y_1 e y_2 são LI

Portanto $y = c_1 e^t + c_2 te^t$ formam uma solução ger[al]

Solução

2c) $y_1 = x$, $y_1' = 1$, $y_1'' = 0$

$x^2y'' - x(x+2)y' + (x+2)y = 0$ (Subst)

$0 - x(x+2)1 + x(x+2) = 0$

$0 = 0$ y_1 é solução!

$y_2 = xe^x$, $y_2' = e^x + xe^x$, $y_2'' = 2e^x + xe^x$

$x^2(2e^x + xe^x) - x(x+2)(e^x + xe^x) + (x+2)xe^x = 0$ (Subst)

$2x^2e^x + x^3e^x - x^3e^x - 2x^2e^x = 0$

$0 = 0$ y_2 é solução

$\frac{y_1}{y_2} = \frac{x}{xe^x} = e^{-x} \neq cte$ logo y_1 e y_2 são LI

Portanto, $y = c_1x + c_2xe^x$ formam uma solução geral!

3a) $2y'' + 5y' + 3y = 0$ $y(0) = 3$ $y'(0) = -4$ Reais e distintas

$a = 2$, $b = 5$, $c = 3$

$ar^2 + br + c = 0$

$2r^2 + 5r + 3 = 0$

$r = \frac{-5 \pm \sqrt{25-24}}{4}$ → -1, $-3/2$

$y = c_1e^{-x} + c_2e^{-3/2x}$

$y' = -c_1e^{-x} - \frac{3}{2}c_2e^{-3/2x}$

$y(0) = c_1 + c_2 = 3$

$y'(0) = -c_1 - \frac{3c_2}{2} = -4$

$c_1 = 1$

$c_2 = 2$

$\boxed{y(x) = e^{-x} + 2e^{-3/2x}}$

3b) $y'' + 3y = 0$ $y(0) = 1$; $y'(0) = 3$

$a = 1$, $b = 0$, $c = 3$

$ar^2 + br + c = 0$

$r^2 + 3 = 0$ ∴ $r = \pm\sqrt{3}i$

Complexas $\alpha = 0$ $\beta = \sqrt{3}$

$y(x) = e^{\alpha x}[C_1\cos(\beta x) + C_2\text{sen}(\beta x)]$

$y(x) = C_1\cos(\sqrt{3}x) + C_2\text{sen}(\sqrt{3}x)$

$y'(x) = -\sqrt{3}C_1\text{sen}(\sqrt{3}x) + \sqrt{3}C_2\cos(\sqrt{3}x)$

c) e d) Usar o mesmo procedimento

4) $m = 2kg$

$R = 14 kg/s$

$F = kx$ ∴ $k = \frac{F}{x} = \frac{6}{0{,}5} = 12\frac{N}{m}$

$x(0) = 1 m$

$x'(0) = 0 m/s$

$2x'' + 14x' + 12x = 0$

$2r^2 + 14r + 12 = 0$

$r = \frac{-14 \pm \sqrt{196-96}}{4}$ → -1, -6 → $x(t) = c_1e^{-t} + c_2e^{-6t}$

$x'(t) = -c_1e^{-t} - 6c_2e^{-6t}$

utilizando as CIs

$x(0) = c_1 + c_2 = 1$

$x'(0) = -c_1 - 6c_2 = 0$

$-5c_2 = 1$

$c_2 = -1/5$ $c_1 = 1 + \frac{1}{5} = \frac{5+1}{5} = \frac{6}{5}$

$x(t) = \frac{6}{5}e^{-t} - \frac{1}{5}e^{-6t}$

5) a)

i) $x^2y'' + 2xy' - 1 = 0$ $\quad v = y' \quad v' = y''$

$x^2v' + 2xv - 1 = 0$

$v' + \frac{2v}{x} = \frac{1}{x^2} \quad p(x) = \frac{2}{x} \quad \mu(x) = e^{\int \frac{2}{x}dx} = e^{\ln x^2} = x^2$

$\cdot\mu(x)$

$x^2v' + 2xv = 1$

$\frac{d}{dx}(x^2v) = 1 \to \int d(x^2v) = \int dx \quad v = y'$

$x^2v = x + C_1 \therefore v = \frac{1}{x} + \frac{C_1}{x^2}$

$y' = \frac{1}{x} + \frac{C_1}{x^2} \therefore \int dy = \int \left(\frac{1}{x} + \frac{C_1}{x^2}\right)dx$

$\boxed{y = \ln|x| - C_1x^{-1} + C_2}$

ii) e iii) mesmo procedimento.

5b)

i) $yy'' + (y')^2 = 0$ $\quad v = y' \quad \frac{dv}{dx} = \frac{dv}{dy}\frac{dy}{dx}$

Subst. em i $\quad y'' = \frac{dv}{dx} = \frac{dv}{dy}v$

$yv\frac{dv}{dy} + v^2 = 0$

$\frac{dv}{dy} + \frac{v}{y} = 0 \therefore \frac{dv}{dy} = -\frac{v}{y} \therefore \int \frac{dv}{v} = -\int \frac{dy}{y}$

$\ln|v| = \ln|y^{-1}| + C_1 \therefore v = e^{\ln y^{-1}}e^{C_1} = y^{-1}C_2$

Como $v = y'$

$y' = C_2y^{-1} \therefore \int y\,dy = \int C_2\,dx \therefore \frac{y^2}{2} = C_2x + C_3$

$y^2 = 2C_2x + 2C_3 \therefore \boxed{y = \sqrt{C_4x + C_5}}$

Mesmo procedimento para ii e iii

Soluções Lista 5

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Solução

Lista 5

1) a) $y'' + y' - 2y = 2t \qquad y(0) = 0 \qquad y'(0) = 1$

Solução $y = y_c + y_p$

Solução y_c

$$y'' + y' - 2y = 0 \quad \left.\begin{matrix} a = 1 \\ b = 1 \\ c = -2 \end{matrix}\right\} \quad \begin{matrix} r^2 + r - 2 = 0 \\ p = -2 \quad r_1 = -2 \\ S = -1 \quad r_2 = 1 \end{matrix}$$

$$y_c = c_1 e^{-2t} + c_2 e^{t}$$

Solução y_p

$y'' + y' - 2y = 2t$ → para esta função obtém-se da tabela $y_p = At + B$. Como esta solução não faz parte da solução y_c não é necessário $\cdot t$.

$$\left.\begin{matrix} y_p = At + B \\ y'_p = A \\ y''_p = 0 \end{matrix}\right\} \quad \begin{matrix} y'' + y' - 2y = 2t \\ 0 + A - 2(At + B) = 2t \\ -2At + (A - 2B) = 2t \end{matrix}$$

$$-2A = 2 \therefore A = -1 \qquad A - 2B = 0 \therefore B = -\tfrac{1}{2} \qquad \boxed{y_p = -t - \frac{1}{2}}$$

$$y = c_1 e^{-2t} + c_2 e^{t} - t - \frac{1}{2} \rightarrow y(0) = c_1 + c_2 - \tfrac{1}{2} = 0$$

$$y' = -2c_1 e^{-2t} + c_2 e^{t} - 1 \qquad y'(0) = -2c_1 + c_2 - 1 = 1 \quad \left.\right\} \begin{matrix} c_1 = -\tfrac{1}{2} \\ c_2 = 1 \end{matrix}$$

$$\boxed{y = -\frac{1}{2} e^{-2t} + e^{t} - t - \frac{1}{2}}$$

Mesmo procedimento para os demais.

2) Usar o Maple ou Mathematica

$y'' + 3y' = 2t^4 + t^2 e^{-3t} + \text{sen}(3t) \rightarrow$

3) $y'' - 5y' + 6y = 2e^{t}$ (I) $\quad \left.\right] \quad y'' - 5y' + 6y = 0 \quad \left\{\begin{matrix} a = 1 \\ b = -5 \\ c = 6 \end{matrix}\right.$

$y_p = u_1 y_1 + u_2 y_2$

$r^2 - 5r + 6 = 0 \quad \left\{\begin{matrix} r_1 = 3 \\ r_2 = 2 \end{matrix}\right.$

- $y_p = u_1 e^{3t} + u_2 e^{2t}$

$y_1 = e^{3t} \qquad y_2 = e^{2t}$

- $y'_p = (u_1' e^{3t} + u_2' e^{2t}) + (3u_1 e^{3t} + 2u_2 e^{2t})$

$\rightarrow u_1' e^{3t} + u_2' e^{2t} = 0$ [Imposição] (II)

- $y''_p = (3u_1' e^{3t} + 2u_2' e^{2t}) + (9u_1 e^{3t} + 4u_2 e^{2t})$

Substituindo y_p e suas derivadas em (I)

$$[(3u_1' e^{3t} + 2u_2' e^{2t}) + (\cancel{9u_1 e^{3t}} + \cancel{4u_2 e^{2t}})] - 5[\cancel{3u_1 e^{3t}} + \cancel{2u_2 e^{2t}}] + 6[\cancel{u_1 e^{3t}} + \cancel{u_2 e^{2t}}] =$$

$$= 2e^{t} \quad \text{(III)}$$

$u_1' e^{3t} = 2e^{t}$

$u_1' = 2e^{2t}$

Resolvendo II e III

$$u_1' e^{3t} + u_2' e^{2t} = 0 \quad \therefore u_2' e^{2t} = -u_1' e^{3t}$$

$$u_1' 3e^{3t} + u_2' 2e^{2t} = 2e^{t} \rightarrow u_1' 3e^{3t} - 2u_1' e^{3t} = 2e^{t}$$

$$\boxed{u_1' = 2e^{-2t}} \quad \therefore \quad \boxed{u_2' = \frac{-2e^{-2t} e^{3t}}{e^{2t}} = -2e^{-t}}$$

$$u_1' = 2e^{-2t} \rightarrow u_1 = \int u_1' dt = \int 2e^{-2t} dt = -e^{-2t} + C_1$$

$$u_2' = -2e^{-t} \rightarrow u_2 = \int u_2' dt = \int -2e^{-t} dt = 2e^{-t} + C_2$$

$e^{2t} \quad u = 2t \quad du = 2 \quad dt = \frac{du}{2}$

Solução

Como buscamos uma solução particular as duas constantes anteriores são igualadas a zero

Assim

$$y_p = u_1 e^{3t} + u_2 e^{2t} = -e^{-2t} e^{3t} + 2e^{-t} e^{2t} = -e^{t} + 2e^{t} = e^{t}$$

$$y = y_c + y_p = C_1 e^{3t} + C_2 e^{2t} + e^{t}$$

Mesma lógica para os demais

1. Resolva o sistema formado pelas eqs. II e III
2. Obtenha u_1 e u_2 a partir de u_1' e u_2'.
3. Obtenha y_p e finalmente $y = y_c + y_p$

4. Aplicar o método de coeficientes indeterminados na edo linear de 2ª ordem não homogênea

$$mx'' + kx = F_0 \cos(\omega_0 t) \qquad \omega = \sqrt{\frac{k}{m}}$$

$a = m$

$b = 0$

$c = k$

$$y = y_c + y_p$$

$$mr^2 + k = 0$$

$$r = \pm\sqrt{-\frac{k}{m}} = \pm\sqrt{\frac{k}{m}}\, i = \pm \omega i \rightarrow y_c = c_1 \cos(\omega t) + c_2 \operatorname{sen}(\omega t)$$

$$y_p = A\cos(\omega_0 t) + B \operatorname{sen}(\omega_0 t) \rightarrow \text{Seguir o método}$$

5)

a) $ay'' + by' + cy = 0$

b) $ar^2 + br + c = 0$

c) Caso I: Raízes reais distintas $\quad y = c_1 e^{r_1 t} + c_2 e^{r_2 t}$

Caso II: Raízes reais idênticas $\quad y = c_1 e^{rt} + c_2 t e^{rt}$

Caso III: Raízes complexas $\alpha \pm i\beta$ $\quad y = e^{\alpha t}[c_1 \cos(\beta t) + c_2 \operatorname{sen}(\beta t)]$

d) Problema composto pela EDO + $y(0) = h$; $y'(0) = j$

e) Problema composto pela EDO + $y(p_1) = h$; $y(p_2) = j$

f) $ay'' + by' + cy = g(t)$

g) A equação complementar equivale a EDO homogênea associada. Ela permite determinar a parte y_c que forma a resposta $y = y_c + y_p$

h) Consultar material de aula.

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